Piecewise linear actions and Zimmer's program

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Abstract

We consider Zimmer's program of lattice actions on surfaces by PL homomorphisms. It is proved that when the surface is not the torus or Klein bottle the action of any finite-index subgroup of $\mathrm{SL}(n,\mathbb{Z})$, $n \geq 5$ (more generally for any 2-big lattice), factors through a finite group action. The proof is based on an establishment of a PL version of Reeb-Thurston's stability.

1 Introduction

This paper introduces a new approach to study lattice actions on compact manifolds.

The special linear group $\mathrm{SL}_n(\mathbb{Z})$ acts on the spheres S^{n-1} by $x \longmapsto Ax/\|x\|$ for $x \in S^{n-1}$ and $A \in \mathrm{SL}_n(\mathbb{Z})$, i.e. via the action on the space of infinite rays based at $0 \in \mathbb{R}^n$. It is believed that this action is minimal in the following sense.

Conjecture 1.1 *Any action of a finite-index subgroup of $\mathrm{SL}_n(\mathbb{Z})$ ($n \geq 3$) on a compact r-manifold by homeomorphisms factors through a finite group action if $r < n - 1$.*

The smooth version of such conjecture was formulated by Farb and Shalen [5], which is related to Zimmer's program [18]. Conjecture 1.1 could be a special case of a general conjecture in Zimmer's program, in which the special linear group is replaced by an arbitrary irreducible lattice Γ in a semisimple Lie group G of $\mathbb{R}$-rank at least 2, and the integer n is replaced by a suitable integer $h(G)$. Some form of Conjecture 1.1 has been discussed by Weinberger [16]. These conjectures are part of a program to generalize the Margulis Superrigidity Theorem to a nonlinear context.

Even for the smooth case, it is difficult to prove this conjecture. In general, we have to assume either that the group action preserves additional geometric structures or that the lattices themselves are special. The following is an incomplete list of some results in this direction. For more details of Zimmer's program and related topics, see survey articles of Zimmer and Morris [20], Fisler [6] and Labourie [10].

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When $r = 1$ and $M = S^1$, Witte [14] proves that Conjecture 1.1 is true for an arithmetic lattice Γ with $\mathbb{Q}$-rank$(\Gamma) \geq 2$.

When $r = 2$, the group action is smooth real-analytic and M is a compact surface other than the torus or Klein bottle, Farb and Shalen [5] prove that Conjecture 1.1 is true for $n \geq 5$ (more generally for 2-big lattices). When the group action is smooth real-analytic and volume-preserving, they also show that this result could be extended to all compact surfaces.

Polterovich (see Corollary 1.1.D of [12]) proves that if $n \geq 3$, then any action by $\mathrm{SL}(n, \mathbb{Z})$ on a closed surface other than the sphere S^2 and the torus T^2 by area preserving diffeomorphisms factors through a finite group action. When $r = 2$ and the group action is by area preserving diffeomorphisms, Franks and Handel [7] prove that Conjecture 1.1 is true for an almost simple group containing a subgroup isomorphic to the three-dimensional integer Heisenberg group (eg. any finite-index subgroup of $\mathrm{SL}_n(\mathbb{Z})$ for $n \geq 3$).

Weinberger [15] shows that Conjecture 1.1 is true for $\mathrm{SL}_n(\mathbb{Z})$ itself and $M = T^r$, the torus of dimension r. Bridson and Vogtmann [1] prove Conjecture 1.1 for $\mathrm{SL}_n(\mathbb{Z})$ and $M = S^r$, the sphere of dimension r.

In this article, we consider lattice actions on manifolds by PL homeomorphisms. Let's recall from Farb and Shalen [5] that an (uniform or non-uniform) irreducible lattice Γ in a semisimple group G of $\mathbb{R}$-rank at least 2 is 1-big if either $\mathbb{Q}$-rank$(\Gamma) \geq 2$ or the centralizer of some infinite-order element of Γ has a subgroup isomorphic to an irreducible lattice in some semisimple Lie group of $\mathbb{R}$-rank at least 2. A lattice is $(k+1)$-big if the centralizer of some infinite-order element has a subgroup isomorphic to a k-big lattice. For example, any finite-index subgroup of $\mathrm{SL}_n(\mathbb{Z})$ is k-big for $n \geq 2k+1$. Our main result is the following.

Theorem 1.2 *Let Γ be a 2-big lattice and M a compact surface (with or without boundary) other than the torus or Klein bottle. Then any PL action of Γ on M factors through a finite group action.*

The proof of Theorem 1.2 consists of two steps, similar to those of some other results of this kind. The first step is to obtain a global fixed point. The second step is to show that the action is trivial for a subgroup of finite index in Γ. In the smooth real-analytic case, there are also two such steps essentially (cf. Farb and Shalen [5]). For the first step, it is achieved by studying the analytic set of the fixed point set of a map and for the second step it is easy. For smooth action, the second step is normally achieved by Reeb-Thurston's stability. Based on such stability, Zimmer [19] shows that the general version of Conjecture 1.1 is true if Γ has a global fixed point. The work of Farb and Shalen [5] is an important motivation for this work, and the strategies of proofs in [5] often play a role in several of our arguments in this short article.

Compared with real-analytic actions, the first step is easier for PL actions, since fixed point sets are simplicial complexes. However, the second step is much more difficult, since the Reeb-Thurston stability is not available for PL actions as it is only valid for C^1 actions. Therefore, we have to establish a PL version

of Reeb-Thurston's stability. For this, we introduce a notion of tangent spheres for PL manifolds (for details, see Definition 3.2), which plays the same role as tangent spaces for smooth manifolds.

Theorem 1.3 *Let G be a finitely generated group acting on a connected manifold M by PL homeomorphisms with a global fixed point p. Suppose that*

(i) G acts trivially on the tangent sphere R_p;

(ii) the first homology group $H_1(G; \mathbb{R}) = 0$,

then the group action is trivial, i.e. G fixes every point of M.

For group actions on 3-manifolds, we obtain the following.

Theorem 1.4 *Let Γ be a 2-big lattice and M be a 3-manifold whose boundary has nonzero Euler characteristic. Then any PL action of Γ on M factors through a finite group action.*

2 k-big lattices

Recall the definition of k-big lattices from the previous section. The following are some typical examples of k-big lattices (cf. [5], Section 2):

- Any finite-index subgroup of $\mathrm{SL}_n(\mathbb{Z})$ is k-big if $n \geq 2k + 1$.

- Let Γ be a lattice in a semisimple Lie group whose root system is not D_4. Γ is k-big if $\mathbb{Q}$-rank of Γ is at least $2k$.

- Let K be a finite real extension of $\mathbb{Q}$ and Φ be a nondegenerate quadratic form of type (p, q) over K. If Φ is diagonal and $(p, q) \geq (3k + 3, 2k + 2)$, the group of K-integral unimodular matrices preserving Φ is k-big.

A group is *almost simple* if every normal subgroup is either finite and central, or is of finite index. The Margulis Finiteness Theorem (cf. [17], Theorem 8.1) implies that a k-big lattice Γ is almost simple. Therefore, the normal subgroup of any noncentral element in Γ is of finite index. We will use such fact several times in later arguments.

3 A PL version of Reeb-Thurston's stability

The following Reeb-Thurston's stability (cf. [13]) says that for a smooth group action if the group G acts trivially on the tangent space of a global fixed point and there is no nontrivial homomorphism from G to the real number $\mathbb{R}$, then the group action is trivial.

Lemma 3.1 (Reeb-Thurston Stability Theorem) *Suppose that the group G is finitely generated and acts by C^1 diffeomorphisms on a connected manifold M, with a fixed point p. If*

(i) G acts trivially on the tangent space $T_p(M)$, and

(ii) $H_1(G; \mathbb{R}) = 0$,

> *then the action is trivial (i.e., every point of M is fixed by every element of G).*

This is a useful lemma to tell when a group action is (globally) trivial in terms of local information of the action.

In this section, we will prove a PL version of the Reeb-Thurston stability. Let's recall some basic facts on PL homeomorphisms. Let M^n be a manifold (with or without boundary) embedded into some Euclidean space. A homeomorphism f of M is piecewise linear (or PL) if there is some triangulation of M into finitely many simplices such that f is affine linear when restricted to each simplex in the triangulation. The composition of two PL homeomorphisms is again a PL homeomorphism by subdivisions of triangulations. Let $\text{Homeo}_{\text{PL}}(M)$ be the set of all PL homeomorphisms of M. A group G action on M by PL homeomorphisms means a group homomorphism $G \to \text{Homeo}_{\text{PL}}(M)$.

Since there is no standard concepts of tangent spaces for PL manifolds, we define a notion of tangent spheres (for our purpose) as an analogy for tangent spaces, as follows.

Definition 3.2 *Let M^n be a manifold with a triangulation Σ. Suppose that p is a vertex in Σ. The tangent sphere R_Σ at p is the set of all rays with root p in the star of p.*

The topology of tangent spheres is described in the following lemma. Note that tangent spheres are not always spheres.

Lemma 3.3 *Define a topology on the tangent sphere R_Σ as the one induced from polar coordinates. Then when $p \in M^n \backslash \partial M^n$, R_Σ is homeomorphic to S^{n-1}, the sphere of dimension $n-1$; when $p \in \partial M$, R_Σ is homeomorphic to D^{n-1}, a disk in S^{n-1} which is bounded by a finite number of hyperplanes passing through the origin.*

Proof. This is obvious by noting that we could choose a unit vector in each ray. ∎

We consider group actions on tangent spheres. Let G act on M^n by PL homeomorphisms. Suppose that p is a global fixed point. For each element $g \in G$, there is a triangulation Σ_g (with p as a vertex) on which g acts simplicially. Without loss of generality, we can fix a chart of p and view p as a point in a Euclidean space. Since the action of g is PL, the tangent sphere R_{Σ_g} is invariant under this action. If $\Sigma_{g'}$ is a subdivision of Σ_g, there is a bijection $i_{g,g'} : R_{\Sigma_g} \to R_{\Sigma_{g'}}$ by restrictions. The map $i_{g,g'}$ is a homeomorphism.

Suppose that B is a symmetric generating set ($g \in B$ implies $g^{-1} \in B$) of a finitely generated group G. We also assume that B contains the identity 1. For each positive integer i, denote by B^i the set of products of i elements in B and Σ_{B^i} the triangulation generated by all Σ_g for $g \in B^i$. We see that Σ_{B^i} is a subdivision of Σ_{B^j} for any $j < i$. Similarly, we can define $R_{\Sigma_{B^i}}$. Denote by Σ_G the direct limit $\varinjlim_{i \to +\infty} \Sigma_{B^i}$ and R_p the direct limit of $\varinjlim_{i \to +\infty} R_{\Sigma_{B^i}}$. We call R_p the tangent sphere of M at p with respect to the group action of G and a symmetric generating set B.

Lemma 3.4 *Let a finitely generated group G act on a manifold M by PL homeomorphisms with a global fixed point p. There is an induced action of G on the tangent sphere R_p by PL homeomorphisms.*

Proof. It is straightforward that the bijection $R_{\Sigma_{B^i}} \to R_{\Sigma_{B^{i+1}}}$ is compactible with actions of B^i. Therefore, there is a group action of G on R_p induced from the action on M. For each element $g \in G$, $g \in B^i$ for some integer i. By construction, each element of B^i acts on $R_{\Sigma_{B^i}} \cong R_p$ piecewise linearly. ∎

Remark 3.5 *It is not hard to see that the definition of R_p is independent of choices of generating sets.*

Note that R_p is homeomorphic to S^{n-1} when $p \notin \partial M$ or D^{n-1} when $p \in \partial M$. In order to prove Theorem 1.3, we need the following two lemmas.

Lemma 3.6 *Let G be a finitely generated group and $H_1(G; \mathbb{R}) = 0$. Then any action of G on $[0, 1)$ (resp. $[0, 1]$) by PL homeomorphisms fixing 0 (resp. $\{0, 1\}$) is trivial.*

Proof. Since 0 is a global fixed point, we could define a map $G \to \mathbb{R}^*$ by

$$g \mapsto (dg)(0),$$

the one-sided derivative of g at 0. This is a group homomorphism. Therefore, a commutator h in G will be a constant function in a neighborhood $[0, a_h]$ of 0. Since $H_1(G; \mathbb{R}) = 0$ and G is finitely generated, $H_1(G; \mathbb{Z})$ is finite and the commutator subgroup $[G, G]$ is finitely generated. Suppose that elements $h_1, h_2, \cdots, h_k$ generate $[G, G]$ and they together with other elements $s_1, s_2, \cdots, s_m$ generate the whole group G. There is a triangulation of $[0, 1)$ on which each element of $\{h_1, h_2, \cdots, h_k, s_1, s_2, \cdots, s_m\}$ acts piecewise linearly. Let a be the minimum positive vertex in the 0-skeleton of this triangulation. Without loss of generality, assume that $\{h_1, h_2, \cdots, h_k, s_1, s_2, \cdots, s_m\}$ is symmetric and contains the identity. Then $[G, G]$ acts trivially on $[0, a]$. Since $H_1(G; \mathbb{Z})$ is finite and a group acting effectively on $[0, a]$ must be left orderable, the action of $\{s_1, s_2, \cdots, s_m\}$ is also trivial on $[0, a]$. Therefore, the action of G on $[0, a]$ is trivial. We claim that there is no largest such $a < 1$, which means the action on $[0, 1)$ of G is trivial. On the contrary, suppose that a_0 is the largest such a. Then the group action of G on $[a, 1)$ (resp. $[a, 1]$) is piecewise linear with

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a a global fixed point. Repeat previous argument, we could find a new $a' > a$ such that G acts trivially on $[0, a']$. This is a contradiction by the assumption of a. ∎

The following lemma is a high-dimensional analogy of the previous lemma.

Lemma 3.7 *Let G be a finitely generated group and $H_1(G; \mathbb{R}) = 0$ and L a polytope. Suppose that G acts on L by PL homeomorphisms with its boundary fixed. Then the action of G on R_p is trivial for any vertex $p \in \partial L$.*

Proof. The idea of the proof is similar to that of the previous one. Choose a vertex p of L and consider the set of faces $\{F_1, F_2, \cdots, F_m\}$ containing p. For each such face F_i, we define a map $G \to \mathbb{R}^{n-1} \backslash \{0\}$ (n is the dimension of L) by

$$g \mapsto (dg)|_{F_i}(p),$$

the directional derivative of p along F_i. Since G acts trivially on F_i, this is a group homomorphism. Then every element in the commutator subgroup will have image $(1, 1, \ldots, 1)$. This implies that a commutator acts trivially on a neighborhood of p. Therefore, the action of G on R_p factors through $H_1(G; \mathbb{Z})$, which is a finite abelian group (since $H_1(G; \mathbb{R}) = 0$). However, any homeomorphism of a disk with boundary fixed must be torsion-free (cf. [16], p. 264). This implies that G acts trivially on R_p. ∎

In the following, we prove the PL version of Reeb-Thurston's stability.
Proof of Theorem 1.3. Let B be a finite symmetric generating set containing the identity of G. Since G acts trivially on the tangent sphere R_p, each ray in R_{Σ_B} is invariant by the action of B. Therefore, each element $g \in B$ maps a ray $r_p \in R_{\Sigma_B}$ to itself and so does each $g \in G$. Fix any ray r_p, which is homeomorphic to $[0, 1)$. Then the group G acts piecewise linearly on r_p. By Lemma 3.6, G acts trivially on r_p. Therefore, G acts trivially on the star St_p of p in the triangulation Σ_B. Choose another vertex p' in the boundary of the star of p. By Lemma 3.7, the group action of G on $(\overline{M \backslash \mathrm{St}_p}) \cap R_{p'}$ is trivial and so is on $R_{p'}$. Repeating the argument as above, this shows that the fixed point set $\mathrm{Fix}(G)$ is both open and closed, which implies that $\mathrm{Fix}(G)$ is the whole manifold M. ∎

4 Actions on circles, surfaces and 3-manifolds

We need a fixed-point result of Fuller [8]:

Lemma 4.1 *Let M be an orientable compact combinatorial manifold and $f : M \to M$ be a homeomorphism. If the Euler characteristic of M is not zero, then f has a periodic point, i.e. $f^k(x) = x$ for some integer k and some $x \in M$.*

The following lemma gives a description of the fixed point set of a PL homeomorphism.

Lemma 4.2 *Suppose that M is a compact manifold. Let $f : M \to M$ be a PL homeomorphism with nonempty fixed point set $\mathrm{Fix}(f)$. Then*

(i) $\mathrm{Fix}(f)$ *is a finite simplicial complex;*

(ii) *there is a (not necessarily connected) closed manifold $N_f \subset \mathrm{Fix}(f)$ (of codimension at least 1 if $f \neq \mathrm{id}_M$) such that for any PL homeomorphism $g : M \to M$ with $g(\mathrm{Fix}(f)) = \mathrm{Fix}(f)$, we have $g(N_f) = N_f$.*

Proof. Since f is affine linear at each simplex σ, the intersection $\mathrm{Fix}(f) \cap \sigma$ is also a simplicial complex (as a solution of a linear equation). Since M is compact, $F = \mathrm{Fix}(f)$ is a union of finite simplicial complexes and therefore a simplicial complex. If $f \neq \mathrm{id}_M$, $F \neq M$. Since F is closed and compact, the boundary $\partial\mathrm{Fix}(f) = F \backslash \mathring{F}$ is a manifold of codimension at least 1, which is invariant under the action of g. If $\partial\mathrm{Fix}(f)$ is a closed manifold, take $N_f = \partial\mathrm{Fix}(f)$. Otherwise, take $N_f = \partial(\partial\mathrm{Fix}(f))$. ∎

Witte [14] proves that a lattice Γ of $\mathbb{Q}$-rank at least 2 acts trivially on the circle S^1 by homeomorphisms. Ghys [9] and Burger-Monod [2] show that any lattice acts trivially on S^1 by C^1 homeomorphisms. The following result considers PL actions for 1-big lattices.

Theorem 4.3 *Let Γ be a 1-big lattice. Then any action of Γ on S^1 by PL homeomorphisms factors through a finite group action.*

Proof. Let $\phi : \Gamma \to \mathrm{Home}_{\mathrm{PL}}(S^1)$ be a group homomorphism. If the $\mathbb{Q}$-rank of Γ is at least 2, Witte [14] shows that Γ acts trivially on S^1. Hence, by the definition of 1-big, we may assume that there is an element $\gamma \in \Gamma$ such that the centralizer $C_\Gamma(\gamma)$ contains a subgroup Λ isomorphic to an irreducible lattice in some semisimple Lie group of $\mathbb{R}$-rank at least 2. By passing to a subgroup of index 2 if necessary, we may assume that the action is orientation-preserving.

Suppose that $\tilde{\gamma} : \mathbb{R} \to \mathbb{R}$ is a lifting of $\phi(\gamma)$. Define the *rotation number* of $\phi(\gamma)$ as

$$\alpha = \lim_{n \to +\infty} \gamma^n(x)/n.$$

Such number is independent of x (cf. Theorem 1, p. 74 in [3]). Moreover, α is rational if and only if $\phi(\gamma)^k$ has a fixed point for some integer k. We prove the theorem in two cases.

(1) α is rational.

This means that the fixed point set $\mathrm{Fix}(\phi(\gamma^k))$ is not empty. If $\gamma^k \in \ker\phi$, the normal subgroup generated by γ^k in Γ is of finite index. Therefore, ϕ factors through a finite group. If $\phi(\gamma^k) \neq \mathrm{id}_{S^1}$, Lemma 4.2 implies that there is a manifold N of codimension at least 1 in $\mathrm{Fix}(\phi(\gamma^k))$ and N is invariant under the action of Λ. Since S^1 is compact, N has only finitely many components, i.e. N is a set of finitely many points. Then for some finite-index subgroup Λ_0 of Λ, Λ_0 has a global fixed point. Since $\Lambda_0/[\Lambda_0, \Lambda_0]$ is finite and the group

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action is orientation preserving, the action Λ_0 is trivial by the PL version of Reeb-Thurston's stability (cf. Theorem 1.3). Note that the normal subgroup generated by Λ_0 in Γ is of finite index. Therefore, ϕ factors through a finite group.

(2) α is irrational.

The proof is similar to that of Theorem 1.1 in Farb and Shalen [5]. We briefly repeat it here for completeness. The Denjoy Theorem (cf. [11], p.38) says that there is an element $\rho \in \text{Home}(S^1)$ such that $T_\alpha := \rho\phi(\gamma)\rho^{-1}$ is the rotation by angle α. The cyclic group $\langle T_\alpha \rangle$ generated by T_α is dense in the group R of rotations of S^1, since α is irrational. Then the subgroup $\rho\phi(\Lambda)\rho^{-1}$ commutes with $\langle T_\alpha \rangle$ and thus commutes with R. Since a homeomorphism commuting with any rotation must be a rotation, $\rho\phi(\Lambda)\rho^{-1}$ is a subgroup of R. Therefore, there is a noncentral element $\gamma' \in \Lambda$ satisfying $\phi(\gamma') = \text{id}_{S^1}$. The normal subgroup generating by γ' is of finite index and therefore ϕ factors through a finite group. ∎

Proof of Theorem 1.2. Suppose that $\phi : \Gamma \to \text{Home}_{\text{PL}}(M)$ is a group homomorphism. First, let us consider the case when M is a closed surface other than the torus or Klein bottle. We will show that the image $\text{Im}\,\phi$ is finite. Suppose that there is an element $\gamma \in \Gamma$ such that the centralizer $C_\Gamma(\gamma)$ contains a subgroup Λ isomorphic to a 1-big irreducible lattice in some semisimple Lie group. Denote by $f = \phi(\gamma)$. Since the Euler characteristic of M is nonzero, Lemma 4.1 implies that for some integer k, the fixed point set $\text{Fix}(f^k)$ is not empty. Then the subgroup Λ acts on $\text{Fix}(f^k)$. There two cases to consider.

(1) $\text{Fix}(f^k) = M$.

The element γ^k acts trivially on M, i.e. $\gamma^k \in \ker\phi$. The normal subgroup generated by γ^k is of finite index by Magulis finiteness theorem. This shows that $\text{Im}\,\phi$ is finite.

(2) $\text{Fix}(f^k) \neq M$.

Lemma 4.2 implies that there is a manifold N of codimension at least 1 in $\text{Fix}(\phi(\gamma^k))$ and N is invariant under the action of Λ. If $\dim N = 1$, N consists of finitely many copies of S^1. Therefore, some subgroup Λ_0 of finite index in Λ will act on a S^1. By Theorem 4.3, there exists a finite-index normal subgroup Λ' of Λ_0 acting trivially on $S^1 \subset N$. Since the Λ' is 1-big, Λ' acts trivially on the tangent sphere S^1 of a point $p \in S^1 \subset N$ by Theorem 4.3 once again. Note that $H_1(\Lambda'; \mathbb{R}) = 0$. The PL version of Reeb-Thurston's stability (cf. Theorem 1.3) implies that the subgroup Λ' acts trivially on M. The normal subgroup generated by Λ' in Γ is of finite index. Therefore $\text{Im}\,\phi$ is finite. The case of $\dim N = 0$ could be considered similarly.

If the boundary $\partial M \neq \varnothing$, ∂M is a disjoint union of finitely many circles (since M is compact). It is clear that Γ maps ∂M to itself. Therefore for

some finite-index subgroup Γ_0 of Γ, a component S^1 of ∂M is invariant under the action of Γ_0. Since Γ_0 is also a 2-big lattice, there exists an infinite-order element $\gamma' \in \Gamma_0$ such that the centralizer $C_{\Gamma_0}(\gamma')$ contains a subgroup G, which is 1-big. Theorem 4.3 implies that there is a finite-index subgroup G_0 of G acting trivially on S^1. For a point $p \in S^1 \subset \partial M$, the tangent sphere R_p is homeomorphic to the interval $[0,1]$ (cf. Lemma 3.3). By Lemma 3.6, the group G_0 acts trivially on R_p. Similar to case (2), Theorem 1.3 implies that G_0 acts trivially on M. The normal subgroup in Γ generated by G_0 is of finite index. Therefore, $\phi(\Gamma)$ is finite. The proof is finished. ∎

Proof of Theorem 1.4. Since M is compact and the Euler characteristic $\chi(\partial M) \neq 0$, there is a connected closed surface $\Sigma \subset \partial M$ with $\chi(\Sigma) \neq 0$. It is obvious that the boundary ∂M is invariant under the action of Γ. Therefore for some finite-index subgroup Γ_0, Σ is invariant under the action of Γ_0. Since Γ_0 is 2-big, Theorem 1.2 implies that the action is trivial for some finite-index subgroup Λ of Γ_0. It's not hard to see that the tangent sphere R_p for $p \in \Sigma$ is homeomorphic to a disk D^2 (cf. Lemma 3.3), whose boundary is fixed by Λ. By Theorem 1.3 and Lemma 3.6, Λ acts trivially on Σ. Using Theorem 1.3 once again, Λ acts trivially on M. The normal subgroup generated by Λ in Γ is of finite index. Therefore, Γ factors through a finite group. ∎

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